

Limits on the integration constant of the dark radiation term in Brane Cosmology

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Abstract

We consider the constraints from primordial Helium abundances on the constant of integration of the dark radiation term of the brane-world generalized Friedmann equation derived from the Randall-Sundrum Single brane model. We found that – using simple, approximate and semianalytical Method – that the constant of integration is limited to be between -8.9 and 2.2 which limits the possible contribution from dark radiation term to be approximately between -27% to 7% of the background photon energy density.

Although at present a fundamental quantum theory of gravity does not exist, it is generally considered that string theory is the leading candidate. But superstring requires ten extra-dimensions or else bad quantum states become part of the spectrum. The Kaluza – Klein (KK) compactification was invoked to get rid of the superfluous six extra dimensions where they were rolled up into some tiny spaces of their own, until a new picture on extra-dimensions emerged recently, such as in the Horova-Witten eleven-dimensional super gravity. In this context, the ordinary matter fields are not supposed to be defined every where but, in contrast are assumed to be confined in a sub-manifold, called brane, a domain wall, embedded, in a higher dimensional space (The Bulk). Braneworld models are conceptually different from compactified KK models because they don't attempt to derive nongravitational forces from the gravitational oscillations in the extra dimensions, on the contrary, if the extra dimensions are large, the gravitational oscillations have to die out quickly in those directions, so that we can't detect them. There are still KK modes of oscillations in the extra dimensions, but because they couple through gravity, and gravity is mostly confined to the bulk, they are effectively invisible to our world on the brane. In this context an interesting, string inspired proposal was given by Randall and Sundrum [1] where they considered only one extra dimension where our universe is described as a three-brane embedded in a five-dimensional anti-de-Sitter space AdS_5 . In this contest they proposed two models, one contains two branes (RS1) which have equal and opposite tensions, and gives a new approach to the hierarchy

problem and a bulk that contains negative cosmological constant. The other model propose only one brane (RSII), with a positive tension, which may be thought of as arising from sending the negative tension brane off to infinity (For a Review see [2]). In this paper we discuss only the the single brane model. The cosmological evolution of such a brane universe has been extensively investigated and their solutions have been found by several authors [2]-[8]. These solutions reduce to a generalized Freidmann eq. on our brane which can be written as:

$$\left(\frac{\dot{R}}{R}\right)^2 = \left(\frac{K^4}{36}\sigma^2 + \frac{\Lambda_5}{6}\right) + \frac{K^4}{18}\sigma\rho + \frac{K^4}{36}\rho^2 + \frac{E}{R^4} \quad (1)$$

Where σ is the brane tension and Λ_5 is the cosmological constant in the bulk, by fine-tuning the brane tension and the bulk cosmological constant (which expresses the brane cosmological version of the well-known cosmological constant problem) the first term on the right hand side vanishes. The second term can be identified with the density term in the usual Friedmann eq. if we make the substitution $8\pi G = \frac{K^4}{6}\sigma$. For $\rho \ll \sigma$, this term dominates over the quadratic term ρ^2 , which arises from the imposition of a function condition for the scale factor on the surface of the brane, and it would decay rapidly as R^{-8} in the early radiation dominated universe. Hence it is expected to be negligible in the low energy limit, and thus will not be significant during the later nucleosynthesis and photon decoupling epochs of interest here.

The last term which is known as the dark radiation term, and it is derived from the electric part of the five-dimensional Weyl tensor and carries information of the gravitational field outside the brane [9]. The constant E is a constant of integration and mathematically there is some justification for it to be either positive or negative depending on the geometry in the bulk [10]. Our work here is to find the limits on the values of the constant E from primordial nucleosynthesis of helium. The constraints from Big Bang nucleosynthesis on the dark radiation term was investigated in refs. [11] and [12], where further in ref. [12] they included the constraints on the 5-dimensional Planck mass in the quadratic term. However our approach here is different than the one presented in these two references, in that, we use a semianalytical and approximate method with the observational constraints to find the limits on the constant E, while in refes. [11] and [12] they use solely the observational constraints.

Thus with the above considerations, eq.1 becomes (using c. g. s. system):

$$H^2 = \frac{8\pi G}{3}\rho_r + \frac{G\hbar}{c}\frac{E}{R^4} \quad (2)$$

At temperature considered here, we can assume that $RT = \text{Constant}$. We further normalize this constant to equal (in c.g.s system) $\frac{\hbar c}{K}$. Thus eq.(2) becomes:

$$H^2 = \frac{8\pi G}{3}\rho_r + \left(\frac{G\hbar}{c}E\right)\left(\frac{K}{\hbar c}\right)^4 T^4 \quad (3)$$

$$\begin{aligned}
\text{Using } \rho_r &= \left(\frac{ag}{2c^2} \right) T^4 \text{ where } a = \left(\frac{\pi^2 K^4}{15c^3 \hbar^3} \right) \\
\text{therefore } H^2 &= \frac{8\pi G}{3} \rho_r \left(1 + \frac{45}{4\pi^3 g} E \right) \\
&= \frac{8\pi G}{3} \rho_r (1 + 0.03E) \quad , \text{ Using } g = \frac{43}{4}
\end{aligned} \tag{4}$$

The primordial production of ${}^4\text{He}$ is controlled by a competition between the weak interaction rates and the expansion rate of the universe. As long as the weak interaction rates are faster than the expansion rate, the neutron-to-proton ratio $\left(\frac{n}{p} \right)$ tracks its equilibrium value. Eventually as the universe expands and cools, the expansion rate comes to dominate and $\frac{n}{p}$ essentially freezes out at the so called freeze out temperature. However the nucleosynthesis chain which begins with the formation of deuterium through the process $p + n \rightarrow D + \gamma$ is delayed past the point where the temperature has fallen below the deuterium binding energy E_B since there are many photons in the exponential tail of the photon energy distribution with energies $E > E_B$ despite the fact that the average photon energy are less than E_B . (see for example [14]).

Thus the effects of the modification of density in eq.4 is to increase or decrease (depending on the sign of E) the ${}^4\text{He}$ abundance through two factors, one by affecting the value of the freeze out temperature, and the other by modifying the time available for neutron decay. Use of the conservation eq. for radiation with eq.4 leads to:

$$t_m = (1 + 0.03E)^{-\frac{1}{2}} t_s \tag{5}$$

where t_m is the modified time from freeze out to start of nucleosynthesis and t_s is the time calculated from standard model without the extra energy.

The calculation of primordial element abundances is a highly nonlinear problem with many coupled nuclear reactions, and requires a numerical analysis. There are many different numerical codes for doing this calculation starting with Wagoner [15]. They mainly differ in the different factors, they include in their calculations, and particularly the different values they use for the neutron half life [16, 17]. Here we present a simplified and approximate method. In this approximation the primordial helium abundance Y is given by: $\left(x = \frac{N_n}{N_p} \right)$:

$$\begin{aligned}
Y &= \left(\frac{2x}{1+x} \right)_F \exp[-\lambda(t_m - t_F)] \\
&\cong \left(\frac{2x}{1+x} \right)_F \exp[-\lambda t_m]
\end{aligned} \tag{6}$$

Where F represent freeze out and we ignored t_F since it is of order one second. Now using $x = \frac{N_n}{N_p} = \exp\left(-\frac{1.5}{T_{f10}}\right)$ Where T_{f10} is the freeze out temperature $\left(T_n = \frac{T}{(10^n K)} \right)$, and is determined by equating the Hubble constant H to the weak reaction rate η for $n \leftrightarrow p$ conversions. Now

$$H \propto g^{\frac{1}{2}} T_{f10}^2 \quad \text{and} \quad \eta \propto T_{f10}^5$$

so that $T_f^3 \propto g^{\frac{1}{2}}$

Thus, if we assume that the change in density caused by the presence of the dark radiation term is translated into a change in g . Then if $g \rightarrow g + \delta g$, $x \rightarrow x + \delta x$, $Y \rightarrow Y + \delta Y$, thus δY becomes

$$\begin{aligned} \delta Y &= \frac{-x \ell n x e^{-\lambda(1+0.03E)^{-\frac{1}{2}} t_s}}{3(1+x)^2} , \quad \frac{\delta g}{g} \\ \delta Y &= \frac{-x \ell n x e^{-\lambda(1+0.03E)^{-\frac{1}{2}} t_s}}{(3)(1+x)^2} \cdot (0.03) E \end{aligned} \quad (7)$$

Using a predicted value calculated by Lopez and Turner [17], where they found $Yp = 0.246$, and taking a conservative value for the measured value for Y to be

$$0.23 < Y < 0.25$$

Which gives

$$-0.016 \leq \delta Y \leq 0.004$$

If E is positive, then the predicted value will increase, moving closer to the observed high value, thus starting with the positive value, putting $x = 0.14$ (from $Y = 0.246$), $\lambda = 78 \times 10^{-5}$, and $t_s \cong 120$ s, eq.7 becomes

$$\begin{aligned} 0.004 &= (0.002E) \exp \left[-0.094 (1 + 0.03)^{-\frac{1}{2}} \right] \\ &\cong (0.002E) [-0.094 (1 - 0.015E)] \\ &\cong 18.2 \times 10^{-4} (1 + 0.0014E) \\ \text{therefore } E &= 2.2 \end{aligned}$$

For the negative value, with $\delta Y = -0.016$, and noting the limit for negative E set by eq.5, we obtain $E = -8.9$. therefore our main result is: $-8.9 \leq E \leq 2.2$ which limits the possible contribution from dark radiation term to be approximately between 27% and 7% of the background photon energy density. These limits are more restrictive than the values found in ref.[11], particularly for the negative limit. However, we should further point out that the sign of the constant E is not yet a settled issue. In finding the exact cosmological, solutions in the brane world, there were two approaches, one is to describe the 3-brane as a 'domain wall' moving in the five dimensional black-hole geometries [3, 4]. In this case and to avoid naked singularity, the constant E must be negative, and its value is limited by the relation $E \geq -\frac{K^2 \ell^2}{4}$, where K is the curvature, and ℓ is the five dimensional curvature length scale, which defines the five dimensional cosmological constant through the relation $\Lambda_s \equiv -\frac{4}{\ell^2}$. The other approach is by exactly solving the Einstein eqs. In the Gaussian normal coordinate, where in this approach the perturbation analysis constrain E to be positive [5]-[8]. Thus there is no definite answer on the signature of this constant, and some global non-linear analysis will be necessary to resolve this issue.

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References

- [1] Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999); 83, 4690 (1999).
- [2] D. Langlois, Prog. Theor. Phys. Suppl. 148, 181 (2003), P. Brax and C. Van de Bruck, hep-th/0303095, R. Maartens, gr-qc/0312059.
- [3] P. Kraus, J. High Energy Phys. **9912**, 011 (1999).
- [4] D. Ida, J. High Energy Phys., **0009**, 014 (2000).
- [5] S. Mukohyama, Phys. Lett. **B473**, 241 (2000).
- [6] P. Binetruy, C. Deffayet, U. Ellwanger, and D. Langlois, Nucl. Phys. **B565**, 269 (2000). D. Langlois, R. Maartens, M. Sasaki and D. Wands, Phys. Rev. **D532**, 155 (2002).
- [7] D. N. Vollick, Class. Quant. Grav. **18**, 1 (2001).
- [8] A. Hebecker and J. March-Ressell, Nucl. Phys. **B608**, 375 (2001).
- [9] T. Shiromizu, K.I. Maeda, and M. Sasaki, Phys. Rev. **D62**, 024012 (2000)., gr – qc/99/10076.
- [10] S. Mukohyama, [hep-th/9911165], S. Mukohyama, T. Shiromizu and K.I. Maeda, Phys. Rev. **D62**, 024028 (2000); M. Sasaki, T. Shiromizu and K.I. Maeda, Phys. Rev. **D62**, 024008 (2000), hep.th/99/112233.
- [11] K. Ichiki, M. Yahiro, T. Kajino, M. Orito, and G.J. Mathews, [dstro-ph/0203272].
- [12] J. Bratt, A. Gault, R. Scherrer, and T. Walder, [astro-ph/0208133].
- [13] P.J.E. Peebles, Principles of physical cosmology Princeton Univ. Press (Princeton, 1993).
- [14] K. Olive, Nucl. Phys. Proc. Suppl. 70, 521(1999), K. Olive, G. Steigman and T. Walker, Phys. Rep. G. Steigman, astro-ph/0009506 – 333, 389(2000).
- [15] R.V. Wagoner, Ap. J. 179 (1973) 343.
- [16] J. Yang, G. Steigman, D.N. Schramm, and R.T. Rood, Ap. J. 227 (1979), 697, J. Yang et al., Ap. J. 281 (1984) 493, Thomas et al., Ap. J. 406 (1993) 569, P.J. Kernan, Ph. D. Thesis, The Ohio State University (1993).
- [17] R. Lopez and M.S. Turner, Phys. Rev. D59 (1999) 103502.